BARI-TH 516/05
NAPOLI DSF-2005/33

Role of final state interactions in the B meson decay into two pions

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We estimate final state interactions in the B-meson decays into two pions by the Regge model. We consider Pomeron exchange and the leading Regge trajectories that can relate intermediate particles to the final state. In some cases, most notably $B \to \pi^0\pi^0$ and $B \to \pi^+\pi^-$, the effect is relevant and produces a better agreement between theory and experiment.

PACS numbers: 13.25.Hw

I. INTRODUCTION

Measuring the angle α of the unitarity triangle is one of the major challenges of the B-factories BaBar at SLAC, BELLE at KEK and the future LHC at CERN. $B \to \pi\pi$ decay channels were identified long ago as a promising candidate for the extraction of the angle α. Though other channels were subsequently investigated, the B decay into two pions is still object of intense studies, both experimental and theoretical. The task of determining precisely the angle α is complicated by the problem of disentangling two different hadronic matrix elements, each one carrying its own weak phase. They are usually referred to as the tree and penguin contributions.

The theoretical uncertainty in the evaluation of these terms stems from the approximate schemes used to compute the relevant four-quark operators between the hadronic states. In principle one could avoid these uncertainties, fixing all the hadronic parameters by a simultaneous measurement of physical observables. This problem has been addressed by several authors; for example in Ref. [1] the use of isospin symmetry among the various $B \to \pi\pi$ channels was envisaged. This program represents a significant experimental challenge and is therefore useful to have some theoretical indications on the results. In the Standard Model one expects the dominance of tree diagrams in $B \to \pi\pi$ decays, differently from the $B \to \pi K$ decay channels, where penguin contributions should play a key-role. Thus one naively expects that for $B \to \pi\pi$ the hierarchical structure follows the analogous hierarchy of the Wilson coefficients, namely

$$\mathcal{B}(\pi^0\pi^0) \ll \mathcal{B}(\pi^+\pi^-) \approx 2\ \mathcal{B}(\pi^+\pi^0)\ . \tag{1}$$

As discussed in section IV below, present experimental data are at odds with Eq. (1). There can be several factors leading to violations of the expectation (1). First of all the role of penguin operators should not be neglected. Second, one has to go beyond naive factorization and use more sophisticated schemes taking into account QCD in factorization, for example the BBNS approach [2, 3] or the Soft-Collinear-Effective-Theory (SCET) [4–7] (for a recent discussion of B decay into two light mesons in the framework of the SCET see [8]). Finally, (1) does not take into account final state interactions (FSI). To this issue the present paper is devoted. FSI are long distance effects that in some cases might play a significant role; for example in B decays into two light mesons a source of long-distance contributions is provided by the charming penguin diagrams that might produce the discrepancy between experimental data and the naive factorization findings. Charming penguins are contributions where the final state is formed only as an effect of a rescattering process, and is preceded by the formation of an intermediate state containing a $c\bar{c}$ pair [9–14]. In particular for decay channels with a strange light meson in the final state, e.g. $B \to \pi K$, these long-distance contributions are not numerically suppressed. In fact, the Cabibbo–Kobayashi–Maskawa (CKM) matrix elements produce an enhancement $\sim |V_{cb}V^*_{cs}|/|V_{ub}V^*_{us}|$ which can compensate the parametric suppression predicted by QCD factorization.

The role of charming penguins in $B \to \pi\pi$ is less clear. Due to the lack of the above-mentioned enhancement, on general ground one expects a minor role in the $B \to \pi\pi$ decay modes. On the other hand in [8] their role is found to be significant. This matter should be settled, but in any case FSI must be taken into account, be they dominated by charming penguins or by other rescattering processes, involving non-charmed particles in the intermediate state.

The most accurate way to take into account FSI in hadronic B decays is provided by the Regge model of high energy scattering processes, which can be applied to hadronic B decays due to the rather large value of $s = m_B^2$. The advantage of the Regge approach is to evaluate the rescattering not by a Feynman diagram, but by unitarity diagrams and the Watson's theorem [15]. In particular there is no extrapolation of low energy effective theories to the hard momenta regime and therefore no need to introduce arbitrary cutoffs in the light meson momenta, because the high

energy behavior is completely under control. Some studies on the application of the Regge model to B decays are in Refs. [16–18]. Elastic contributions to high energy scattering are dominated by the Pomeron exchange, while the inelastic channels get contributions from both Pomeron and Regge trajectories. Also charming penguins find a place in this scheme, provided one introduces also charmed Regge trajectories, as for example in the study performed in [19] for the charmless B decay into two light vector mesons.

The aim of this letter is to extend the results of the Regge model to the $B \to \pi\pi$ decay modes. We will show that there are indeed significant rescattering effects in the $B \to \pi^0\pi^0$ channel, a decay mode that is suppressed in naive factorization. We include several intermediate states: $\pi\pi$, $\rho\rho$, $a_1\pi$, $D\bar{D}$ and we find that the largest contribution comes from the $\pi\pi$ and $\rho\rho$ intermediate states with ρ and a_2 Regge exchanges respectively. On the other hand we find no significant role of charming penguin contributions. The suppression of charming penguins in comparison to other terms is produced because Regge charmed trajectories have a negative intercept $\alpha(0)$ and therefore a suppression factor $(s/s_0)^{\alpha(0)}$ ($s_0 \simeq 1$ GeV^2, a threshold).

The plan of the paper is as follows. In section II we evaluate in the factorization approximation the bare amplitudes, including tree and penguin contributions with no FSI (in particular no charming penguins). In section III we discuss rescattering effects parameterized by the Regge model. Finally, in section IV we present our numerical results and discuss them.

II. BARE AMPLITUDES

The $\pi\pi$ final state can be reached from several intermediate states *via* rescattering. Clearly one should select the most prominent channels. Among the inelastic channels we single out the decays $B \to \rho\rho$ and $B \to a_1\pi$ since they have large branching ratios. For example: $\mathcal{B}(B^+ \to \rho^0\rho^+) = (26.4 \pm 6.1) \times 10^{-6}$; $\mathcal{B}(B^0 \to \rho^-\rho^+) = (30.0 \pm 6.0) \times 10^{-6}$; $\mathcal{B}(B^0 \to a_1^-\pi^+) = (42.6 \pm 5.9) \times 10^{-6}$ [20]. To these decay processes we have to add the elastic $B \to \pi\pi$ channels, though they have smaller branching ratios. We also add the $D^{(*)}\bar{D}^{(*)}$, having in mind a discussion on the charming penguins. In conclusion the final state interactions that we consider are the elastic scattering $\pi\pi \to \pi\pi$, and the $\rho\rho \to \pi\pi$ and $a_1\pi \to \pi\pi$ and the $D^{(*)}\bar{D}^{(*)} \to \pi\pi$ inelastic channels.

We evaluate bare amplitudes in the factorization approximation. To do that one needs different input parameters. The non-leptonic hamiltonian is well known and we do not repeat it here, see e.g. [21]. For the Wilson coefficients we use: $a_1 = 1.029$, $a_2 = 0.140$, and $(a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}) = (33.33, -246.66, -10, -300, 1.95, 4.81, -93.30, -12.63)$ $\times 10^{-4}$ [22]. We use for the parameterization of the CKM matrix $\sin\theta_{12} = 0.2243$, $\sin\theta_{23} = 0.0413$, $\sin\theta_{13} = 0.0037$ and $\delta_{13} = \gamma = 1.05$ [23]. As for the form factors and constant decay we use $f_\pi = 0.132$ GeV, $f_\rho = 0.210$ GeV, $f_{a_1} \approx 0.21$ GeV (see the discussion in [24]) $F_1^{B\to\pi}(0) = 0.26$, $A_1^{B\to\rho}(0) = 0.26$, $A_2^{B\to\rho}(0) = 0.23$, $V_0^{B\to a_1}(0) = A_0^{B\to\rho}(0) = 0.39$, where we use the notations of [25] for the $B \to \rho$ transition and the parameterization of Ref. [26] for the $B \to a_1\pi$ matrix element. All the other parameters are taken from [23]. We get in this way the results of Tables I-III (notice that units of Tables I-II are 10^{-8} GeV, those of III are 10^{-7} GeV).

Process	A_b	Process	A_b ($\lambda = +1$)	A_b ($\lambda = -1$)	A_b ($\lambda = 0$)
$B^+ \to \pi^+\pi^0$	$+2.02 - 1.24\,i$	$B^+ \to \rho^+\rho^0$	$-0.02 + 0.01\,i$	$-1.1 + 0.65\,i$	$+4.49 - 2.76\,i$
$B^0 \to \pi^0\pi^0$	$-0.41 + 0.053\,i$	$B^0 \to \rho^0\rho^0$	$+0.004 - 0.001\,i$	$+0.20 - 0.07\,i$	$-0.83 + 0.31\,i$
$B^0 \to \pi^+\pi^-$	$+2.43 - 1.74\,i$	$B^0 \to \rho^+\rho^-$	$-0.02 + 0.02\,i$	$-1.31 + 0.85\,i$	$+5.53 - 3.59\,i$

TABLE I: Bare amplitudes for $B \to \pi\pi$ and $B \to \rho\rho$. Results in 10^{-8} GeV; $\lambda = \pm 1, 0$ refers to the helicities of the vector particles.

Process	A_b
$B^+ \to a_1^+\pi^0$	$+3.4 - 2.0\,i$
$B^+ \to a_1^0\pi^+$	$+2.2 - 1.6\,i$
$B^0 \to a_1^0\pi^0$	$-0.60 + 0.20\,i$
$B^0 \to a_1^+\pi^-$	$+4.2 - 2.7\,i$
$B^0 \to a_1^-\pi^+$	$+3.4 - 2.4\,i$

TABLE II: Bare amplitudes for $B \to a_1\pi$; a_1 with longitudinal polarization. Units are 10^{-8} GeV.

Process	A_b	Process	A_b
$B^+ \to D^+ \bar{D}^0$	$-2.8\,i$	$B^+ \to D^{*+} \bar{D}^0$	$+2.5\,i$
$B^0 \to D^+ D^-$	$-2.8\,i$	$B^0 \to D^{*+} D^-$	$+2.5\,i$
$B^+ \to D^{*+} \bar{D}^{*0}$	$-2.9\,i$	$B^+ \to D^+ \bar{D}^{*0}$	$+2.5\,i$
$B^0 \to D^{*+} D^{*-}$	$-2.9\,i$	$B^0 \to D^+ D^{*-}$	$+2.5\,i$

TABLE III: Bare amplitudes $B \to D^{(*)} \bar{D}^{(*)}$; vector particles have longitudinal polarization. Results in 10^{-7} GeV.

III. FINAL STATE INTERACTIONS AND REGGE BEHAVIOR

Corrections to the bare amplitudes due to final state interactions are taken into account by means of the Watson's theorem [15]:

$$A = \sqrt{S} A_b \tag{2}$$

where S is the S-matrix, A_b and A are the bare and the full amplitudes. An application of the Watson's theorem was first discussed in [17] and subsequently applied to other decay channels in [18] and [19]. We briefly review here the formalism.

The two-body S-matrix elements are given by

$$S_{ij}^{(I)} = \delta_{ij} + 2i\sqrt{\rho_i \rho_j}\; T_{ij}^{(I)}(s) \quad , \tag{3}$$

where i, j run over all the channels involved in the final state interactions. The $J = 0$, isospin I amplitude $T_{ij}^{(I)}(s)$ is obtained by projecting the $J = 0$ angular momentum out of the amplitude $T_{ij}^{(I)}(s, t)$:

$$T_{ij}^{(I)}(s) = \frac{1}{16\pi} \frac{s}{\sqrt{\ell_i \ell_j}} \int_{t_+}^{t_-} dt\, T_{ij}^{(I)}(s, t) \quad . \tag{4}$$

$\rho_j\,, \ell_j$ and $t_\pm$ are defined in Ref. [19]. For the channel $B \to \pi\pi$ we only have the $I = 0$ and $I = 2$ transition amplitudes; the decay amplitude $B \to \pi^+\pi^0$ is only $I = 2$.

The phenomenological basis for the application of the Regge model of final state interactions is the large value of $s = m_B^2$; therefore a Regge approximation based on Pomeron exchange and the first leading trajectories should be adequate. The Pomeron term contributes to the elastic channels. As discussed in previous section for the inelastic case we include only channels whose bare amplitudes are prominent.

In conclusion in the present approximation we will include, besides the Pomeron, the ρ and a_2 (almost) exchange-degenerate trajectories and π Regge trajectories. We shall discuss in subsection III B the role of charmed Regge trajectories in parameterizing charming penguins.

For the Pomeron contribution we write (neglecting light meson masses)

$$S \;= 1 + 2iT^{\mathcal{P}}(s)\,, \qquad T^{\mathcal{P}}(s) = \frac{1}{16\pi s} \int_{-s}^{0} T^{\mathcal{P}}(s, t) dt\,, \tag{5}$$

and we use the following parameterization [18, 27]:

$$T^{\mathcal{P}}(s, t) = \; - \; \beta^{\mathcal{P}} g(t) \left(\frac{s}{s_0}\right)^{\alpha_{P(t)}} e^{-i(\pi/2)\alpha_P(t)} \;, \tag{6}$$

with $s_0 = 1\,\mathrm{GeV}^2$ and $\alpha_P(t) = 1.08 + 0.25t$ (t in GeV^2), as given by fits to hadron-hadron scattering total cross sections. For the Pomeron residue $\beta^{\mathcal{P}}$ we assume factorization with a t-dependence given by [18, 27]

$$g(t) = \frac{1}{(1 - t/m_\rho^2)^2} \simeq e^{2.8t} \quad . \tag{7}$$

The additive quark counting rule allows to compute the Pomeron-pion residue in terms of the Pomeron-nucleon ones. This gives [16, 18]:

$$\beta_\pi^{\mathcal{P}} \sim \frac{2}{3} \beta_p^{\mathcal{P}} \sim 5.1 \;. \tag{8}$$

As observed in [17] inelasticity effects play an important role in the determination of the FSI phases. Parameterizing them as in Ref. [17] by one effective state, with no extra phases would allow to write the S-matrix as follows (neglecting a small phase $\varphi = -0.01$ in $\sqrt{1+2iT^{\mathcal{P}}}$):

$$(B \to \pi\pi) \qquad S \approx \begin{pmatrix} 0.62 & 0.82\,i \\ 0.82\,i & 0.62 \end{pmatrix}, \qquad \sqrt{S} \approx \begin{pmatrix} 0.79 & 0.64(1+i) \\ 0.64(1+i) & 0.79 \end{pmatrix}. \tag{9}$$

This shows that even neglecting the effect of the non leading Regge trajectories, final state interactions due to inelastic effects parameterized by the Pomeron exchange can produce sizeable strong phases. This result agrees with the analogous findings of Refs. [17] and [18]. However this method is not useful to evaluate rescattering effect in weak decays. Therefore we prefer to parameterize inelastic effects by Regge trajectories.

A. Regge trajectories

Let us now consider the contribution of the leading Regge trajectories. Including Regge trajectories the S matrix can be written for the generic $B \to \pi\pi$ case as follows

$$S = 1 + 2i\left(T^{\mathcal{P}} + \sum \mathcal{R}\right). \tag{10}$$

Here $\mathcal{P}$ indicates the Pomeron contribution discussed above. Since the Pomeron is much larger than the others we make the approximation

$$A(B \to \pi\pi)^{(I)} \approx \sqrt{1+2iT^{\mathcal{P}}} A_b^{(I)} + \frac{1}{2\sqrt{1+2iT^{\mathcal{P}}}} \sum_k \sum_{\mathcal{R}} (\mathcal{R})^{(k,I)} A_b^{(k)}. \tag{11}$$

Here the sum over k refers to the various intermediate states contributing to the final state $\pi\pi$; I is the isospin index.

We write the Regge amplitudes as follows ($\mathcal{R} = \rho\,, a_2\,, \pi$):

$$\mathcal{R}^{(k,I)}(s) = \frac{1}{16\pi\, s} \int_{-s}^{0} dt \mathcal{R}^{(k,I)}(s,t). \tag{12}$$

We assume the general parameterization

$$\mathcal{R}^{(k,I)}(s,t) \approx -\beta^R \, \frac{1+(-)^{s_R} e^{-i\pi\alpha_R(t)}}{2} \Gamma(l_R - \alpha_R(t)) \; (\alpha')^{1-l_R} \; (\alpha' s)^{\alpha_R(t)} \tag{13}$$

as suggested in Ref. [28]. The trajectory is given by

$$\alpha_R(t) = s_R + \alpha' (t - m_R^2) = \alpha_R(0) + \alpha' t\,, \tag{14}$$

with $\alpha' = 0.91$ GeV^{-2}. We notice the Regge poles at $l_R - \alpha_R(t) = 0, -1, -2, \cdots$. The parameters we use are reported

Trajectory $\mathcal{R}$	s_R	ℓ_R	$\alpha_R(0)$
ρ	1	1	0.5
a_2	2	1	0.5
π	0	0	≈ 0

TABLE IV: Parameters of the Regge trajectories. Exchange degeneracy is assumed.

in Table IV. Near $t = m_R^2$, Eq. (13) reduces to

$$\mathcal{R} \approx \beta^R \frac{s^{s_R}}{(t - m_R^2)}. \tag{15}$$

We write $\beta^R = \beta_1^R \beta_2^R$ using factorization of the residues at the two (1 and 2) vertices. Therefore Eq. (15) allows to identify β^R as the product of two on-shell coupling constants. The residues can be obtained by the decay rates

$\rho \to \pi\pi$, $a_{1,2} \to \rho\pi$. More precisely we obtain $\beta^{\rho}_{\pi^+\pi^0} = 8.2$ and $\beta^{\rho}_{a_1\pi} \approx 2$ from the $\rho \to \pi\pi$ and $a_1 \to \rho\pi$ decay widths, respectively. Due to small value of the residue $\beta^{\rho}_{a_1\pi}$ we will neglect the contribution of this channel in the sequel.

Let us now discuss the a_2 exchange. The residue $\beta^{a_2}_{\rho^+\pi^0}$ can be derived from the strong coupling constant defined by

$$\mathcal{M}(a_2^+(p,\eta) \to \rho^+(k,\epsilon)\pi^0(q)) = \frac{g_a}{m_{a_2}} \eta^{\mu\nu} q_\mu \epsilon_{\nu\alpha\beta\lambda} \epsilon^{*\alpha} p^\beta q^\lambda \,. \tag{16}$$

From $a_2 \to \rho\pi$ we get $g_a \approx 25$ GeV^{-1}. To compute the residue we note that the a_2-exchange can only occur when the ρ intermediate particles have transverse polarization. Its residue is related to g_a by $\beta^{a_2}_{\rho^+\pi^0} = g_a/\sqrt{\alpha'} \approx 13.1$. This phenomenological value is smaller than the theoretical value given in [28], on the basis of the Gell-Mann, Sharp and Wagner model [29] for the $\omega \to 3\pi$ decay. In view of the theoretical uncertainties arising from the hypothesis of exchange-degeneracy and from the procedure we have described, we will let this parameter vary with a spread of $\pm 50\%$ around a central value, i.e. we assume

$$\beta^{a_2}_{\rho^+\pi^0} = 13.1 \times (1 \pm 0.50) \ . \tag{17}$$

We note that, though the bare amplitudes $B \to \rho\rho$ with transversely polarized ρ's are suppressed (see Table I), they participate nevertheless in the rescattering process due to the large residue of the a_2 trajectory to the ρ and π.

As shown by table IV the π trajectory is exponentally suppressed due to $\alpha_\pi \approx 0$. Similarly we note that we have also computed the parameters of the a_1 Regge pole, but we omit this trajectory from the analysis because its intercept is large and negative ($\alpha(0) \approx -0.37$).

B. Charming penguins

Charming penguins are diagrams describing the rescattering of two charmed mesons to produce two light mesons. Treating them as Feynman diagrams produces a huge theoretical uncertainty. In fact to compute them one should employ the chiral effective theory for light and heavy mesons. However this approach cannot be extended to hard meson momenta and one is forced to introduce a cut-off [11–14]. To avoid the arbitrariness of this procedure one can describe this class of FSI by charmed Regge trajectories. This approach was followed in [19] for $B \to \rho\rho$, $K^*\rho$, $K^*\phi$ decays and can be easily extended to $B \to \pi\pi$; we refer to this paper for details. Let us only write down the expression of trajectories $\alpha_D(t)$ and $\alpha_{D^*}(t)$. We use Eq. (14) with $s_D = 0$, $s_{D^*} = 1$ and [19]

$$\alpha_0 = -1.8 \,, \qquad \alpha' = (0.39 \pm 0.12) \text{ GeV}^{-2} \ , \tag{18}$$

which shows that the intercept of these trajectories is negative. Also the residues can be computed following the procedure of previous subsection, i.e. using the strong coupling constants $g_{D^*D\pi}$ and $g_{D^*D^*\pi}$ (for the values of these constants we follow [11]).

Differently from the case of $K\pi$ or the $K^*\rho$ final states, the bare $B \to D^{(*)}\bar{D}^{(*)}$ amplitudes have no CKM enhancement. Therefore the situation is similar to the study of the $B \to \rho\rho$ channel in [19] where we found that, for the $\rho\rho$ final state, charming penguins are less relevant than, for example, $B \to K^*\rho$. We have checked numerically that the negative intercept produces a negligible contribution from the $D^{(*)}\bar{D}^{(*)}$ intermediate states to $B \to \pi\pi$.

IV. NUMERICAL RESULTS AND DISCUSSION

We present our results by taking the γ angle as a parameter and allowing $\beta^{a_2}_{\rho^+\pi^0}$ to vary in the range in Eq. (17). We compute in Fig. 1 the branching ratios $\mathcal{B}(B^0 \to \pi^0\pi^0)$, $\mathcal{B}(B^0 \to \pi^+\pi^-)$ and $\mathcal{B}(B^+ \to \pi^+\pi^0)$. A survey of the experimental results is in table V. Here we have also reported our results for $\beta^{a_2}_{\rho^+\pi^0}$ in the range of values given in Eq. (17). We see that the role of FSI is especially important for the $B \to \pi^0\pi^0$ channel.

Since we do not pretend to have presented a complete discussion of final state interactions, our result should be interpreted as an indication of the relevant role played by the rescattering effects when the bare amplitudes are for some reason small. This is confirmed by the results for the other two channels, where charged current hamiltonian is involved and therefore FSI play a less relevant role. Nevertheless also for the $B^0 \to \pi^+\pi^-$ channel we can see that FSI contribution produce a better agreement with the data,

Process	$\mathcal{B}$ (without FSI)	$\mathcal{B}$ (with FSI)	$\mathcal{B}$ (exp.)
$B^0 \to \pi^0\pi^0$	0.08	$0.10 - 0.65$	$1.17 \pm 0.32 \pm 0.10$ [30, 31]
			$2.3^{+0.4+0.2}_{-0.5-0.3}$ [32, 33]
			1.45 ± 0.29 [20]
$B^0 \to \pi^+\pi^-$	8.1	$3.8 - 4.4$	$4.7 \pm 0.6 \pm 0.2$ [30, 31]
			$4.4 \pm 0.6 \pm 0.3$ [32, 33]
			4.5 ± 0.4 [20]
$B^+ \to \pi^+\pi^0$	5.0	$3.6 - 5.0$	$5.8 \pm 0.6 \pm 0.4$ [30, 31]
			$5.0 \pm 1.2 \pm 0.5$ [32, 33]
			5.5 ± 0.6 [20]

TABLE V: Theoretical branching ratios for $B \to \pi\pi$ decay channels with and without final state interactions and their comparison with experimental data. The column FSI is computed with $\beta^{a_2}_{\rho^+\pi^0}$ in the range given in Eq. (17). Units 10^{-6}.

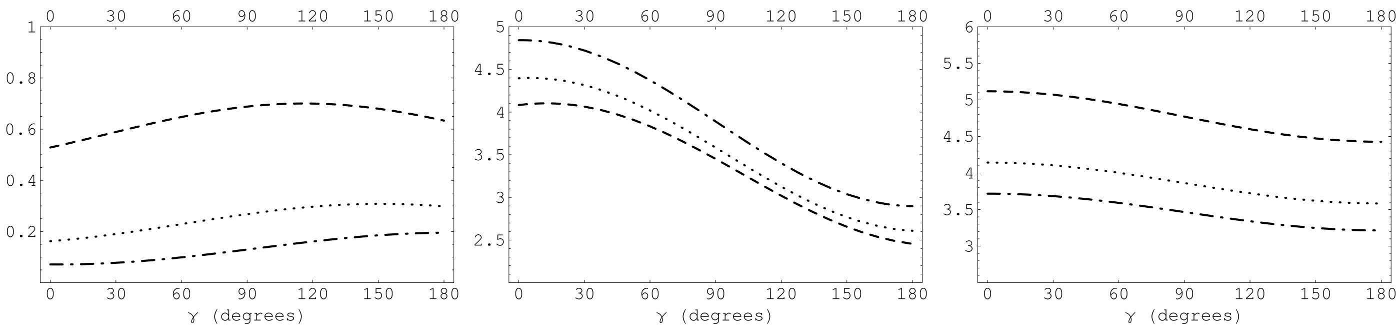

FIG. 1: Branching ratios (units 10^{-6}) for $B \to \pi\pi$ as functions of the angle γ (degrees). From left to right the decays $B^0 \to \pi^0\pi^0$, $B^0 \to \pi^+\pi^-$ and $B^+ \to \pi^+\pi^0$. Dashed, dotted, and dot-dashed lines refer to the upper, central and lower values of the Regge residue in Eq. (17).

We have also computed the integrated asymmetries

$$\begin{aligned}
\mathcal{A}_{00} &= \frac{\Gamma(\bar{B}^0 \to \pi^0\pi^0) - \Gamma(B^0 \to \pi^0\pi^0)}{\Gamma(\bar{B}^0 \to \pi^0\pi^0) + \Gamma(B^0 \to \pi^0\pi^0)} \,, \\
\mathcal{A}_{+-} &= \frac{\Gamma(\bar{B}^0 \to \pi^+\pi^-) - \Gamma(B^0 \to \pi^+\pi^-)}{\Gamma(\bar{B}^0 \to \pi^+\pi^-) + \Gamma(B^0 \to \pi^+\pi^-)} \,, \\
\mathcal{A}_{-0} &= \frac{\Gamma(B^- \to \pi^-\pi^0) - \Gamma(B^+ \to \pi^+\pi^0)}{\Gamma(B^- \to \pi^-\pi^0) + \Gamma(B^+ \to \pi^+\pi^0)} \,.
\end{aligned} \tag{19}$$

The results are reported in Fig. 2. For $\mathcal{A}_{00}$ the HFAG group reports the average of the BaBar and Belle Collaborations as follows [20]: $\mathcal{A}_{00} = 0.28 \pm 0.39$. For $\gamma \simeq 60^\circ$ our result is compatible, within error with the experiment.

Let us finally compare our results with other approaches. The exclusive $B \to \pi\pi$ transitions can be studied, starting from first principles, in the *QCD factorization* approach (BBNS) [2, 3]. In this framework, all the charmless two body decays of B mesons have amplitudes which are shown to factorize at lowest order in $1/m_b$. In other words, neglecting terms suppressed by heavy quark mass, the *QCD factorization* predicts the *naive* factorization ansatz [34]. In this framework the branching ratios for the $B \to \pi\pi$ decay modes are rather sensitive to the computational scheme of the relevant form factor. In particular, as discussed in [2], results for the $\pi^0\pi^0$ final state depend on a parameter λ_b whose precise value is unknown, but a branching ratio of the order of 10^{-6} could be reached. In the phenomenological studies [35, 36] of these processes, the authors take into account the power-suppressed and partially unknown weak annihilation contributions. In particular, the last fit to charmless strangeless final state alone in the BBNS approach, performed in Ref. [36], reproduces the available experimental data. Our method complements the BBNS approach as it takes into account in a systematic way part of the power suppressed contributions, i.e. those arising from the final state interactions.

Agreement with experimental data on $B \to \pi\pi$ is also obtained in [8], where the Soft Collinear Effective Theory

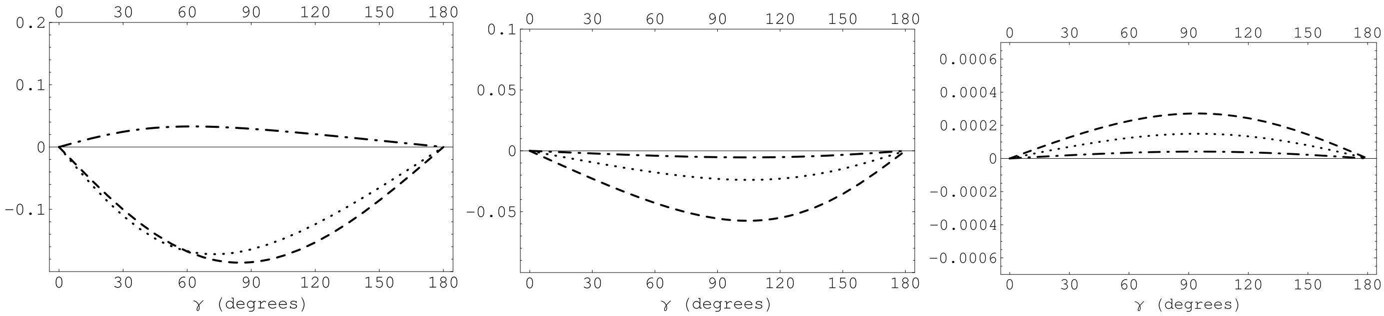

FIG. 2: Time integrated asymmetries as defined in Eqns. (19) as functions of the angle γ (degrees). From the left to right $\mathcal{A}_{00}$, $\mathcal{A}_{+-}$ and $\mathcal{A}_{-0}$. Dashed, dotted, and dot-dashed lines refer to the upper, central and lower values of the Regge residue in Eq. (17).

(SCET) [4–7] is employed. The BBNS and the SCET approaches substantially differ in treating perturbative and non-perturbative effects, in particular SCET predicts non negligible long-distance charming penguin contributions. We do not discuss the differences between QCD-factorization and the SCET as this goes beyond the limits of the present work. We stress however that we do not find an important role of the long distance charming penguin diagrams, but we find another source of long distance effects due to rescattering of the $\pi\pi$ and $\rho\rho$ channels. Charming penguins play a minor role here because, as discussed in section III, in the Regge theory they are strongly suppressed by the negative intercept of the corresponding Regge trajectory. Our results are confirmed by a different analysis [12] of the charming penguin contributions in $B \to \pi\pi$, based on an effective lagrangian approach; also in that paper these contributions play a lesser role the reason being, there as in the present work, the absence of any CKM enhancement in the bare amplitudes.

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